Background knowledge

Contents:

This chapter contains material that is normally covered prior to this course. It is assumed knowledge. Not all preliminaries are covered within the chapter. However, other necessary work is revised within the chapters.

A SURDS AND RADICALS

Real numbers like $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, etc., are called **surds** or **radicals**. Surds are present in solutions to some quadratic equations. $\sqrt{4}$ is a radical but is not a surd as it simplifies to 2. Surds are irrational real numbers.

Definition: $\sqrt{a}$ is the non-negative number such that $\boldsymbol{\sqrt{a} \times \sqrt{a} = a}$.

Properties:

- $\sqrt{a}$ is never negative, so $\boldsymbol{\sqrt{a} \geqslant 0}$.
- $\sqrt{a}$ is meaningful only for $a \geqslant 0$.
- $\boldsymbol{\sqrt{ab} = \sqrt{a} \times \sqrt{b}}$ for $a \geqslant 0$ and $b \geqslant 0$.
- $\boldsymbol{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}}$ for $a \geqslant 0$ and $b > 0$.

EXERCISE A

Example 1

Write as a single surd: **a** $\sqrt{2} \times \sqrt{3}$ **b** $\frac{\sqrt{18}}{\sqrt{6}}$

a $\sqrt{2} \times \sqrt{3}$
$= \sqrt{2 \times 3}$
$= \sqrt{6}$

b $\frac{\sqrt{18}}{\sqrt{6}}$
$= \sqrt{\frac{18}{6}}$
$= \sqrt{3}$

or $\frac{\sqrt{18}}{\sqrt{6}}$
$= \frac{\sqrt{6} \times \sqrt{3}}{\sqrt{6}}$
$= \sqrt{3}$

1 Write as a single surd or rational number:

a $\sqrt{3} \times \sqrt{5}$ **b** $(\sqrt{3})^2$ **c** $2\sqrt{2} \times \sqrt{2}$ **d** $3\sqrt{2} \times 2\sqrt{2}$

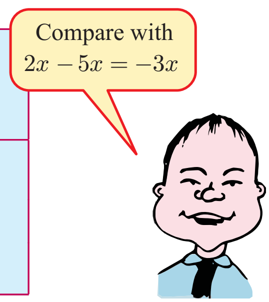
e $3\sqrt{7} \times 2\sqrt{7}$ **f** $\frac{\sqrt{12}}{\sqrt{2}}$ **g** $\frac{\sqrt{12}}{\sqrt{6}}$ **h** $\frac{\sqrt{18}}{\sqrt{3}}$

Example 2

Simplify: **a** $3\sqrt{3} + 5\sqrt{3}$ **b** $2\sqrt{2} - 5\sqrt{2}$

a $3\sqrt{3} + 5\sqrt{3}$
$= (3 + 5)\sqrt{3}$
$= 8\sqrt{3}$

b $2\sqrt{2} - 5\sqrt{2}$
$= (2 - 5)\sqrt{2}$
$= -3\sqrt{2}$



2 Simplify the following mentally:

a $2\sqrt{2}+3\sqrt{2}$ **b** $2\sqrt{2}-3\sqrt{2}$ **c** $5\sqrt{5}-3\sqrt{5}$ **d** $5\sqrt{5}+3\sqrt{5}$

e $3\sqrt{5}-5\sqrt{5}$ **f** $7\sqrt{3}+2\sqrt{3}$ **g** $9\sqrt{6}-12\sqrt{6}$ **h** $\sqrt{2}+\sqrt{2}+\sqrt{2}$

Example 3

Write $\sqrt{18}$ in the form $a\sqrt{b}$ where a and b are integers and a is as large as possible.

$$\begin{aligned} &\sqrt{18} \\ &= \sqrt{9\times 2} \quad \{9 \text{ is the largest perfect square factor of } 18\} \\ &= \sqrt{9}\times\sqrt{2} \\ &= 3\sqrt{2} \end{aligned}$$

3 Write the following in the form $a\sqrt{b}$ where a and b are integers and a is as large as possible:

a $\sqrt{8}$ **b** $\sqrt{12}$ **c** $\sqrt{20}$ **d** $\sqrt{32}$

e $\sqrt{27}$ **f** $\sqrt{45}$ **g** $\sqrt{48}$ **h** $\sqrt{54}$

i $\sqrt{50}$ **j** $\sqrt{80}$ **k** $\sqrt{96}$ **l** $\sqrt{108}$

Example 4

Simplify: $2\sqrt{75}-5\sqrt{27}$

$$\begin{aligned} &2\sqrt{75}-5\sqrt{27} \\ &= 2\sqrt{25\times 3} - 5\sqrt{9\times 3} \\ &= 2\times 5\times\sqrt{3} - 5\times 3\times\sqrt{3} \\ &= 10\sqrt{3}-15\sqrt{3} \\ &= -5\sqrt{3} \end{aligned}$$

4 Simplify:

a $4\sqrt{3}-\sqrt{12}$ **b** $3\sqrt{2}+\sqrt{50}$ **c** $3\sqrt{6}+\sqrt{24}$

d $2\sqrt{27}+2\sqrt{12}$ **e** $\sqrt{75}-\sqrt{12}$ **f** $\sqrt{2}+\sqrt{8}-\sqrt{32}$

Example 5

Write $\frac{9}{\sqrt{3}}$ without a radical in the denominator.

$$\begin{aligned} &\frac{9}{\sqrt{3}} \\ &= \frac{9}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{9\sqrt{3}}{3} \\ &= 3\sqrt{3} \end{aligned}$$

5 Write without a radical in the denominator:

a $\frac{1}{\sqrt{2}}$ **b** $\frac{6}{\sqrt{3}}$ **c** $\frac{7}{\sqrt{2}}$ **d** $\frac{10}{\sqrt{5}}$ **e** $\frac{10}{\sqrt{2}}$

f $\frac{18}{\sqrt{6}}$ **g** $\frac{12}{\sqrt{3}}$ **h** $\frac{5}{\sqrt{7}}$ **i** $\frac{14}{\sqrt{7}}$ **j** $\frac{2\sqrt{3}}{\sqrt{2}}$

B SCIENTIFIC NOTATION (STANDARD FORM)

Scientific notation (or **standard form**) involves writing any given number as a number between 1 and 10, multiplied by a power of 10,

i.e., $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$.

EXERCISE B

Example 6

Write in standard form: **a** 37 600 **b** 0.000 86

a $37\,600 = 3.76 \times 10\,000$ {shift decimal point 4 places to the
$= 3.76 \times 10^4$ left and $\times$ 10 000}

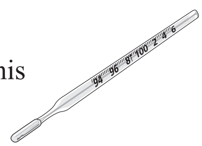
b $0.000\,86 = 8.6 \div 10^4$ {shift decimal point 4 places to the
$= 8.6 \times 10^{-4}$ right and $\div$ 10 000}

1 Express the following in scientific notation:

a 259 **b** 259 000 **c** 2.59 **d** 0.259
e 0.000 259 **f** 40.7 **g** 4070 **h** 0.0407
i 407 000 **j** 407 000 000 **k** 0.000 040 7

2 Express the following in scientific notation:

a The distance from the Earth to the Sun is 149 500 000 000 m.
b Bacteria are single cell organisms, some of which have a diameter of 0.0003 mm.
c A speck of dust is smaller than 0.001 mm.
d The core temperature of the Sun is 15 million degrees Celsius.
e A single red blood cell lives for about four months and during this time it will circulate around the body 300 000 times.



Example 7

Write as an ordinary number:
a 3.2×10^2 **b** 5.76×10^{-5}

a 3.2×10^2
$= 3.20 \times 100$
$= 320$

b 5.76×10^{-5}
$= 000005.76 \div 10^5$
$= 0.000\,057\,6$

3 Write as an ordinary decimal number:

a 4×10^3 **b** 5×10^2 **c** 2.1×10^3 **d** 7.8×10^4
e 3.8×10^5 **f** 8.6×10^1 **g** 4.33×10^7 **h** 6×10^7

4 Write as an ordinary decimal number:

a 4×10^{-3} **b** 5×10^{-2} **c** 2.1×10^{-3} **d** 7.8×10^{-4}
e 3.8×10^{-5} **f** 8.6×10^{-1} **g** 4.33×10^{-7} **h** 6×10^{-7}

5 Write as an ordinary decimal number:

a The wavelength of light is 9×10^{-7} m.
b The estimated world population for the year 2000 was 6.130×10^9.
c The diameter of our galaxy, the Milky Way, is 1×10^5 light years.
d The smallest viruses are 1×10^{-5} mm in size.

6 Find, correct to 2 decimal places:

a $(3.42 \times 10^5) \times (4.8 \times 10^4)$ **b** $(6.42 \times 10^{-2})^2$ **c** $\dfrac{3.16 \times 10^{-10}}{6 \times 10^7}$
d $(9.8 \times 10^{-4}) \div (7.2 \times 10^{-6})$ **e** $\dfrac{1}{3.8 \times 10^5}$ **f** $(1.2 \times 10^3)^3$

7 If a missile travels at 5400 km h^{-1} how far will it travel in:

a 1 day **b** 1 week **c** 2 years?

Give your answers in scientific notation correct to 2 decimal places, and assume that 1 year $\approx$ 365.25 days.

8 Light travels at a speed of 3×10^8 metres per second. How far will light travel in:

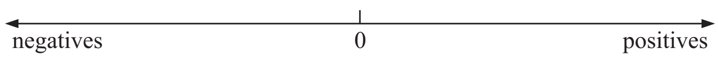
a 1 minute **b** 1 day **c** 1 year?

Give your answers with decimal part correct to 2 decimal places, and assume that 1 year $\approx$ 365.25 days.

C NUMBER SYSTEMS AND SET NOTATION

NUMBER SYSTEMS

We use:
- $\mathbb{R}$ to represent the set of all **real numbers**. These include all of the numbers on the number line.



- $\mathbb{N}$ to represent the set of all **natural numbers**. $\mathbb{N} = \{0, 1, 2, 3, 4, 5, ...\}$
- $\mathbb{Z}$ to represent the set of all **integers**. $\mathbb{Z} = \{0, \pm1, \pm2, \pm3, \pm4, ...\}$
- $\mathbb{Z}^+$ is the set of all positive integers. $\mathbb{Z}^+ = \{1, 2, 3, 4, ...\}$
- $\mathbb{Q}$ to represent the set of all **rational numbers** which are any numbers of the form $\dfrac{p}{q}$ where p and q are integers, $q \neq 0$.

SET NOTATION

$\{x \mid -3 < x < 2\}$ reads "the set of all values that x can be such that x lies between -3 and 2".

the set of all — *such that*

Unless stated otherwise, we assume that x is real.

$-3 < x < 2$ can also be written as $x \in]-3, 2[$.

EXERCISE C

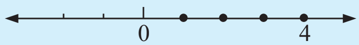
1 Write verbal statements for the meaning of:

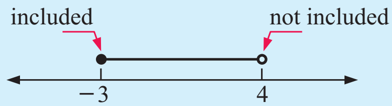
a $\{x \mid x > 5, \quad x \in \mathbb{R}\}$ **b** $\{x \mid x \leqslant 3, \quad x \in \mathbb{Z}\}$ **c** $\{y \mid 0 < y < 6\}$

d $\{x \mid 2 \leqslant x \leqslant 4, \quad x \in \mathbb{Z}\}$ **e** $\{t \mid 1 < t < 5\}$ **f** $\{n \mid n < 2 \text{ or } n \geqslant 6\}$

Example 8

Write in set notation:

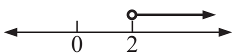
a 

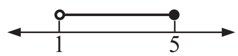
b 

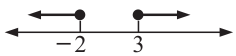
a $\{x \mid 1 \leqslant x \leqslant 4, \quad x \in \mathbb{N}\}$
or $\{x \mid 1 \leqslant x \leqslant 4, \quad x \in \mathbb{Z}\}$

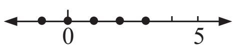
b $\{x \mid -3 \leqslant x < 4, \quad x \in \mathbb{Z}\}$

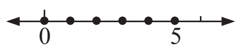
2 Write in set notation:

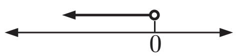
a 

b 

c 

d 

e 

f 

3 Sketch the following number sets:

a $\{x \mid 4 \leqslant x < 10, \quad x \in \mathbb{N}\}$ **b** $\{x \mid -4 < x \leqslant 5, \quad x \in \mathbb{Z}\}$

c $\{x \mid -5 < x \leqslant 4, \quad x \in \mathbb{R}\}$ **d** $\{x \mid x > -4, \quad x \in \mathbb{Z}\}$

e $\{x \mid x \leqslant 8, \quad x \in \mathbb{R}\}$

D ALGEBRAIC SIMPLIFICATION

To answer the following questions, you will need to remember:

- the distributive law $\boldsymbol{a(b+c) = ab + ac}$
- power notation $a^2 = a \times a, \quad a^3 = a \times a \times a, \quad a^4 = a \times a \times a \times a,$ and so on.

EXERCISE D

1 Simplify if possible:

a $3x + 7x - 10$ **b** $3x + 7x - x$ **c** $2x + 3x + 5y$

d $8 - 6x - 2x$ **e** $7ab + 5ba$ **f** $3x^2 + 7x^3$

2 Remove the brackets and then simplify:

a $3(2x+5)+4(5+4x)$ **b** $6-2(3x-5)$

c $5(2a-3b)-6(a-2b)$ **d** $3x(x^2-7x+3)-(1-2x-5x^2)$

3 Simplify:

a $2x(3x)^2$ **b** $\dfrac{3a^2b^3}{9ab^4}$ **c** $\sqrt{16x^4}$ **d** $(2a^2)^3\times 3a^4$

E LINEAR EQUATIONS AND INEQUALITIES

When dealing with inequalities:

- multiplying or dividing both sides by a negative reverses the inequality sign.
- do not multiply or divide both sides by the unknown or a term involving the unknown.

EXERCISE E

1 Solve for x:

a $2x+5=25$ **b** $3x-7>11$ **c** $5x+16=20$

d $\dfrac{x}{3}-7=10$ **e** $6x+11<4x-9$ **f** $\dfrac{3x-2}{5}=8$

g $1-2x\geqslant 19$ **h** $\frac{1}{2}x+1=\frac{2}{3}x-2$ **i** $\frac{2}{3}-\dfrac{3x}{4}=\frac{1}{2}(2x-1)$

2 Solve simultaneously for x and y:

a $\begin{cases} x+2y=9 \\ x-y=3 \end{cases}$ **b** $\begin{cases} 2x+5y=28 \\ x-2y=2 \end{cases}$ **c** $\begin{cases} 7x+2y=-4 \\ 3x+4y=14 \end{cases}$

d $\begin{cases} 5x-4y=27 \\ 3x+2y=9 \end{cases}$ **e** $\begin{cases} x+2y=5 \\ 2x+4y=1 \end{cases}$ **f** $\begin{cases} \dfrac{x}{2}+\dfrac{y}{3}=5 \\ \dfrac{x}{3}+\dfrac{y}{4}=1 \end{cases}$

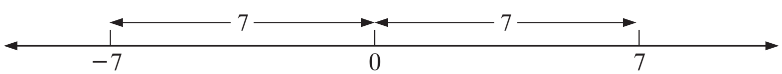
F MODULUS OR ABSOLUTE VALUE

The **modulus** or **absolute value** of a real number is its size, ignoring its sign.

For example: the modulus (or absolute value) of 7 is 7, and
the modulus (or absolute value) of -7 is also 7.

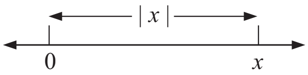
GEOMETRIC DEFINITION

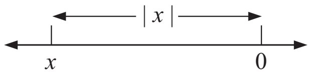
The modulus of a real number is its *distance* from zero on the number line. Because the modulus is a distance, it cannot be negative.



We denote the modulus of x as $|x|$.

$|x|$ is the distance of x from 0 on the real number line.

If $x > 0$ 

If $x < 0$ 

$|x - a|$ can be considered as 'the distance of x from a'.

ALGEBRAIC DEFINITION

$$|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases} \quad \text{or} \quad |x| = \sqrt{x^2}$$

MODULUS EQUATIONS

It is clear that $|x| = 2$ has two solutions, $x = 2$ and $x = -2$, since $|2| = 2$ and $|-2| = 2$.

If $|x| = a$ where $a > 0$, then $x = \pm a$.

EXERCISE F

1 Find the value of:

a $5 - (-11)$ **b** $|5| - |-11|$ **c** $|5 - (-11)|$

d $\left|(-2)^2 + 11(-2)\right|$ **e** $|-6| - |-8|$ **f** $|-6 - (-8)|$

2 If $a = -2$, $b = 3$, and $c = -4$ find the value of:

a $|a|$ **b** $|b|$ **c** $|a|\,|b|$ **d** $|ab|$

e $|a - b|$ **f** $|a| - |b|$ **g** $|a + b|$ **h** $|a| + |b|$

i $|a|^2$ **j** a^2 **k** $\left|\dfrac{c}{a}\right|$ **l** $\dfrac{|c|}{|a|}$

3 Solve for x:

a $|x| = 3$ **b** $|x| = -5$ **c** $|x| = 0$

d $|x - 1| = 3$ **e** $|3 - x| = 4$ **f** $|x + 5| = -1$

g $|3x - 2| = 1$ **h** $|3 - 2x| = 3$ **i** $|2 - 5x| = 12$

G PRODUCT EXPANSION

$y = 2(x - 1)(x + 3)$ can be expanded into the general form $y = ax^2 + bx + c$.

Likewise, $y = 2(x - 3)^2 + 7$ can also be expanded into this form.

Following is a **list of expansion rules** you can use:

- $(a + b)(c + d) = ac + ad + bc + bd$
- $(a + b)(a - b) = a^2 - b^2$ {**difference of two squares**}
- $(a + b)^2 = a^2 + 2ab + b^2$ {**perfect squares**}

EXERCISE G

Example 9

Expand and simplify:

a $(2x+1)(x+3)$ **b** $(3x-2)(x+3)$

a $(2x+1)(x+3)$
$= 2x^2 + 6x + x + 3$
$= 2x^2 + 7x + 3$

b $(3x-2)(x+3)$
$= 3x^2 + 9x - 2x - 6$
$= 3x^2 + 7x - 6$

1 Expand and simplify using $(a+b)(c+d) = ac + ad + bc + bd$:

a $(2x+3)(x+1)$ **b** $(3x+4)(x+2)$ **c** $(5x-2)(2x+1)$
d $(x+2)(3x-5)$ **e** $(7-2x)(2+3x)$ **f** $(1-3x)(5+2x)$
g $(3x+4)(5x-3)$ **h** $(1-3x)(2-5x)$ **i** $(7-x)(3-2x)$
j $(5-2x)(3-2x)$ **k** $-(x+1)(x+2)$ **l** $-2(x-1)(2x+3)$

Example 10

Expand using the rule $(a+b)(a-b) = a^2 - b^2$:

a $(5x-2)(5x+2)$ **b** $(7+2x)(7-2x)$

a $(5x-2)(5x+2)$
$= (5x)^2 - 2^2$
$= 25x^2 - 4$

b $(7+2x)(7-2x)$
$= 7^2 - (2x)^2$
$= 49 - 4x^2$

2 Expand using the rule $(a+b)(a-b) = a^2 - b^2$:

a $(x+6)(x-6)$ **b** $(x+8)(x-8)$ **c** $(2x-1)(2x+1)$
d $(3x-2)(3x+2)$ **e** $(4x+5)(4x-5)$ **f** $(5x-3)(5x+3)$
g $(3-x)(3+x)$ **h** $(7-x)(7+x)$ **i** $(7+2x)(7-2x)$
j $(x+\sqrt{2})(x-\sqrt{2})$ **k** $(x+\sqrt{5})(x-\sqrt{5})$ **l** $(2x-\sqrt{3})(2x+\sqrt{3})$

Example 11

Expand using the perfect square expansion rule:

a $(x+2)^2$ **b** $(3x-1)^2$

a $(x+2)^2$
$= x^2 + 2(x)(2) + 2^2$
$= x^2 + 4x + 4$

b $(3x-1)^2$
$= (3x)^2 + 2(3x)(-1) + (-1)^2$
$= 9x^2 - 6x + 1$

3 Expand and simplify using the perfect square expansion rule:

a $(x+5)^2$ **b** $(x+7)^2$ **c** $(x-2)^2$ **d** $(x-6)^2$
e $(3+x)^2$ **f** $(5+x)^2$ **g** $(11-x)^2$ **h** $(10-x)^2$
i $(2x+7)^2$ **j** $(3x+2)^2$ **k** $(5-2x)^2$ **l** $(7-3x)^2$

4 Expand the following into the general form $y = ax^2 + bx + c$:

a $y = 2(x+2)(x+3)$ **b** $y = 3(x-1)^2 + 4$ **c** $y = -(x+1)(x-7)$

d $y = -(x+2)^2 - 11$ **e** $y = 4(x-1)(x-5)$ **f** $y = -\frac{1}{2}(x+4)^2 - 6$

g $y = -5(x-1)(x-6)$ **h** $y = \frac{1}{2}(x+2)^2 - 6$ **i** $y = -\frac{5}{2}(x-4)^2$

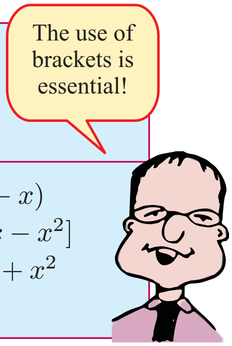
Example 12

Expand and simplify:

a $1 - 2(x+3)^2$ **b** $2(3+x) - (2+x)(3-x)$

a
$$\begin{aligned} & 1 - 2(x+3)^2 \\ &= 1 - 2[x^2 + 6x + 9] \\ &= 1 - 2x^2 - 12x - 18 \\ &= -2x^2 - 12x - 17 \end{aligned}$$

b
$$\begin{aligned} & 2(3+x) - (2+x)(3-x) \\ &= 6 + 2x - [6 - 2x + 3x - x^2] \\ &= 6 + 2x - 6 + 2x - 3x + x^2 \\ &= x^2 + x \end{aligned}$$



5 Expand and simplify:

a $1 + 2(x+3)^2$ **b** $2 + 3(x-2)(x+3)$

c $3 - (3-x)^2$ **d** $5 - (x+5)(x-4)$

e $1 + 2(4-x)^2$ **f** $x^2 - 3x - (x+2)(x-2)$

g $(x+2)^2 - (x+1)(x-4)$ **h** $(2x+3)^2 + 3(x+1)^2$

i $x^2 + 3x - 2(x-4)^2$ **j** $(3x-2)^2 - 2(x+1)^2$

H FACTORISATION

Algebraic **factorisation** is the reverse process of expansion.

For example, $(2x+1)(x-3)$ is **expanded** to $2x^2 - 5x - 3$, whereas
$2x^2 - 5x - 3$ is **factorised** to $(2x+1)(x-3)$.

Notice that $2x^2 - 5x - 3 = (2x+1)(x-3)$ has been factorised into two **linear factors**.

Flow chart for factorising:

Expression

→ Take out any common factors

→ Recognise type

Difference of two squares
$a^2 - b^2 = (a+b)(a-b)$

Perfect square
$a^2 + 2ab + b^2 = (a+b)^2$

Sum and Product type
$\boldsymbol{x^2 + bx + c}$
$x^2 + bx + c = (x+p)(x+q)$
where $p + q = b$ and $pq = c$

Sum and Product type
$\boldsymbol{ax^2 + bx + c, \quad a \neq 0}$
- find ac
- find the factors of ac which add to b
- if these factors are p and q, replace bx by $px + qx$
- complete the factorisation

Example 13

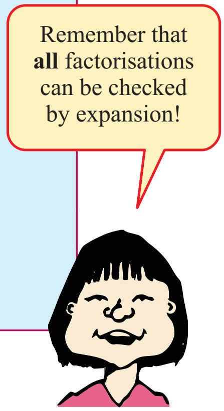
Fully factorise:

a $3x^2 - 12x$ **b** $4x^2 - 1$ **c** $x^2 - 12x + 36$

a $3x^2 - 12x$
$= 3x(x - 4)$ {$3x$ is a common factor}

b $4x^2 - 1$
$= (2x)^2 - 1^2$ {difference of two squares}
$= (2x + 1)(2x - 1)$

c $x^2 - 12x + 36$
$= x^2 + 2(x)(-6) + (-6)^2$ {perfect square form}
$= (x - 6)^2$



EXERCISE H

1 Fully factorise:

a $3x^2 + 9x$ **b** $2x^2 + 7x$ **c** $4x^2 - 10x$
d $6x^2 - 15x$ **e** $9x^2 - 25$ **f** $16x^2 - 1$
g $2x^2 - 8$ **h** $3x^2 - 9$ **i** $4x^2 - 20$
j $x^2 - 8x + 16$ **k** $x^2 - 10x + 25$ **l** $2x^2 - 8x + 8$
m $16x^2 + 40x + 25$ **n** $9x^2 + 12x + 4$ **o** $x^2 - 22x + 121$

Example 14

Fully factorise:

a $3x^2 + 12x + 9$ **b** $-x^2 + 3x + 10$

a $3x^2 + 12x + 9$ {3 is a common factor}
$= 3(x^2 + 4x + 3)$ {sum $= 4$, product $= 3$}
$= 3(x + 1)(x + 3)$

b $-x^2 + 3x + 10$
$= -[x^2 - 3x - 10]$ {removing -1 as a common factor to make the coefficient of x^2 be 1}
$= -(x - 5)(x + 2)$ {sum $= -3$, product $= -10$}

2 Fully factorise:

a $x^2 + 9x + 8$ **b** $x^2 + 7x + 12$ **c** $x^2 - 7x - 18$
d $x^2 + 4x - 21$ **e** $x^2 - 9x + 18$ **f** $x^2 + x - 6$
g $-x^2 + x + 2$ **h** $3x^2 - 42x + 99$ **i** $-2x^2 - 4x - 2$
j $2x^2 + 6x - 20$ **k** $2x^2 - 10x - 48$ **l** $-2x^2 + 14x - 12$
m $-3x^2 + 6x - 3$ **n** $-x^2 - 2x - 1$ **o** $-5x^2 + 10x + 40$

FACTORISATION BY 'SPLITTING' THE x-TERM

Using the distributive law to expand we see that:

$$\begin{aligned}&(2x+3)(4x+5)\\&= 8x^2+10x+12x+15\\&= 8x^2+22x+15\end{aligned}$$

We will now **reverse** the process to **factorise** the quadratic expression $8x^2+22x+15$.

		$8x^2+22x+15$
Step 1:	'Split' the middle term	$= 8x^2+10x+12x+15$
Step 2:	Group in pairs	$= (8x^2+10x)+(12x+15)$
Step 3:	Factorise each pair separately	$= 2x(4x+5)+3(4x+5)$
Step 4:	Factorise fully	$= (4x+5)(2x+3)$

The 'trick' in factorising these types of quadratic expressions is in *Step 1* where the middle term needs to be split into two so that the rest of the factorisation proceeds smoothly.

Rules for splitting the x-term:

The following procedure is recommended for factorising ax^2+bx+c:

- Find ac.
- Find the factors of ac which add to b.
- If these factors are p and q, replace bx by $px+qx$.
- Complete the factorisation.

Example 15

Fully factorise:

a $2x^2-x-10$ **b** $6x^2-25x+14$

a $2x^2-x-10$ has $ac = 2\times -10 = -20$.
The factors of -20 which add to -1 are -5 and $+4$.

$$\begin{aligned}\therefore\ 2x^2-x-10 &= 2x^2-5x+4x-10\\&= x(2x-5)+2(2x-5)\\&= (2x-5)(x+2)\end{aligned}$$

b $6x^2-25x+14$ has $ac = 6\times 14 = 84$.
The factors of 84 which add to -25 are -21 and -4.

$$\begin{aligned}\therefore\ 6x^2-25x+14 &= 6x^2-21x-4x+14\\&= 3x(2x-7)-2(2x-7)\\&= (2x-7)(3x-2)\end{aligned}$$

3 Fully factorise:

a $2x^2 + 5x - 12$	**b** $3x^2 - 5x - 2$	**c** $7x^2 - 9x + 2$
d $6x^2 - x - 2$	**e** $4x^2 - 4x - 3$	**f** $10x^2 - x - 3$
g $2x^2 - 11x - 6$	**h** $3x^2 - 5x - 28$	**i** $8x^2 + 2x - 3$
j $10x^2 - 9x - 9$	**k** $3x^2 + 23x - 8$	**l** $6x^2 + 7x + 2$
m $-4x^2 - 2x + 6$	**n** $12x^2 - 16x - 3$	**o** $-6x^2 - 9x + 42$
p $21x - 10 - 9x^2$	**q** $8x^2 - 6x - 27$	**r** $12x^2 + 13x + 3$
s $12x^2 + 20x + 3$	**t** $15x^2 - 22x + 8$	**u** $14x^2 - 11x - 15$

Example 16

Fully factorise: $3(x+2) + 2(x-1)(x+2) - (x+2)^2$

$3(x+2) + 2(x-1)(x+2) - (x+2)^2$

$= (x+2)[3 + 2(x-1) - (x+2)]$ {as $(x+2)$ is a common factor}

$= (x+2)[3 + 2x - 2 - x - 2]$

$= (x+2)(x-1)$

4 Fully factorise:

a $3(x+4) + 2(x+4)(x-1)$	**b** $8(2-x) - 3(x+1)(2-x)$
c $6(x+2)^2 + 9(x+2)$	**d** $4(x+5) + 8(x+5)^2$
e $(x+2)(x+3) - (x+3)(2-x)$	**f** $(x+3)^2 + 2(x+3) - x(x+3)$
g $5(x-2) - 3(2-x)(x+7)$	**h** $3(1-x) + 2(x+1)(x-1)$

Example 17

Fully factorise using the 'difference of two squares':

a $(x+2)^2 - 9$ **b** $(1-x)^2 - (2x+1)^2$

a $(x+2)^2 - 9$

$= (x+2)^2 - 3^2$

$= [(x+2) + 3][(x+2) - 3]$

$= (x+5)(x-1)$

b $(1-x)^2 - (2x+1)^2$

$= [(1-x) - (2x+1)][(1-x) + (2x+1)]$

$= [1 - x - 2x - 1][1 - x + 2x + 1]$

$= -3x(x+2)$

5 Fully factorise:

a $(x+3)^2 - 16$	**b** $4 - (1-x)^2$	**c** $(x+4)^2 - (x-2)^2$
d $16 - 4(x+2)^2$	**e** $(2x+3)^2 - (x-1)^2$	**f** $(x+h)^2 - x^2$
g $3x^2 - 3(x+2)^2$	**h** $5x^2 - 20(2-x)^2$	**i** $12x^2 - 27(3+x)^2$

INVESTIGATION 1 ANOTHER FACTORISATION TECHNIQUE

What to do:

1 By expanding, show that $\dfrac{(ax+p)(ax+q)}{a} = ax^2 + [p+q]x + \left[\dfrac{pq}{a}\right]$.

2 If $ax^2 + bx + c = \dfrac{(ax+p)(ax+q)}{a}$, show that $p+q=b$ and $pq = ac$.

3 Using **2** on $8x^2 + 22x + 15$, we have

$$8x^2 + 22x + 15 = \frac{(8x+p)(8x+q)}{8} \quad \text{where} \quad \begin{cases} p+q = 22 \\ pq = 8 \times 15 = 120 \end{cases}$$

$\therefore \quad p = 12, \quad q = 10,$ or vice versa

$$\begin{aligned} \therefore \quad 8x^2 + 22x + 15 &= \frac{(8x+12)(8x+10)}{8} \\ &= \frac{\cancel{4}(2x+3)\cancel{2}(4x+5)}{\cancel{8}} \\ &= (2x+3)(4x+5) \end{aligned}$$

a Use the method shown to factorise:

i $3x^2 + 14x + 8$ **ii** $12x^2 + 17x + 6$ **iii** $15x^2 + 14x - 8$

b Check your answers to **a** by expansion.

I FORMULA REARRANGEMENT

In the formula $D = xt + p$ we say that D is the **subject**. This is because D is expressed in terms of the other variables x, t and p.

We can rearrange the formula to make one of the other variables the subject. We do this using the usual rules for solving equations. Whatever we do to one side of the equation we must also do to the other side.

EXERCISE I

Example 18

Make x the subject of $D = xt + p$.

$$\begin{aligned} \text{If} \quad D &= xt + p \\ \therefore \quad xt + p &= D \\ \therefore \quad xt + p - p &= D - p \qquad \{\text{subtract } p \text{ from both sides}\} \\ \therefore \quad xt &= D - p \\ \therefore \quad \frac{xt}{t} &= \frac{D-p}{t} \qquad \{\text{divide both sides by } t\} \\ \therefore \quad x &= \frac{D-p}{t} \end{aligned}$$

1 Make x the subject of:

a $a+x=b$ **b** $ax=b$ **c** $2x+a=d$

d $c+x=t$ **e** $5x+2y=20$ **f** $2x+3y=12$

g $7x+3y=d$ **h** $ax+by=c$ **i** $y=mx+c$

Example 19

Make z the subject of $c=\frac{m}{z}$.

$$c=\frac{m}{z}$$

$$c\times z=\frac{m}{z}\times z \quad \{\text{multiply both sides by } z\}$$

$$\therefore \quad cz=m$$

$$\therefore \quad \frac{cz}{c}=\frac{m}{c} \quad \{\text{divide both sides by } c\}$$

$$\therefore \quad z=\frac{m}{c}$$

2 Make z the subject of:

a $az=\frac{b}{c}$ **b** $\frac{a}{z}=d$ **c** $\frac{3}{d}=\frac{2}{z}$ **d** $\frac{z}{2}=\frac{a}{z}$

3 Make:

a a the subject of $F=ma$ **b** r the subject of $C=2\pi r$

c d the subject of $V=ldh$ **d** K the subject of $A=\frac{b}{K}$.

Example 20

Make t the subject of $s=\frac{1}{2}gt^2$ where $t>0$.

$$\tfrac{1}{2}gt^2=s \quad \{\text{rewrite with } t^2 \text{ on LHS}\}$$

$$\therefore \quad 2\times\tfrac{1}{2}gt^2=2\times s \quad \{\text{multiply both sides by } 2\}$$

$$\therefore \quad gt^2=2s$$

$$\therefore \quad \frac{gt^2}{g}=\frac{2s}{g} \quad \{\text{divide both sides by } g\}$$

$$\therefore \quad t^2=\frac{2s}{g}$$

$$\therefore \quad t=\sqrt{\frac{2s}{g}} \quad \{\text{as } t>0\}$$

4 Make:

a r the subject of $A = \pi r^2$ if $r > 0$

b x the subject of $N = \dfrac{x^5}{a}$

c r the subject of $V = \frac{4}{3}\pi r^3$

d x the subject of $D = \dfrac{n}{x^3}$.

5 Make:

a a the subject of $d = \dfrac{\sqrt{a}}{n}$

b l the subject of $T = \frac{1}{5}\sqrt{l}$

c a the subject of $c = \sqrt{a^2 - b^2}$

d l the subject of $T = 2\pi\sqrt{\dfrac{l}{g}}$

e a the subject of $P = 2(a + b)$

f h the subject of $A = \pi r^2 + 2\pi rh$

g r the subject of $I = \dfrac{E}{R + r}$

h q the subject of $A = \dfrac{B}{p - q}$.

6 **a** Make a the subject of the formula $k = \dfrac{d^2}{2ab}$.

b Find the value for a when $k = 112$, $d = 24$, $b = 2$.

7 The formula for determining the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

a Make r the subject of the formula.

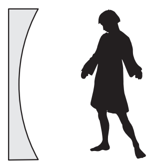
b Find the radius of a sphere which has a volume of 40 cm^3.

8 The distance (in cm) travelled by an object accelerating from a stationary position is given by the formula $S = \frac{1}{2}at^2$ where a is the acceleration in cm s^{-2} and t is the time in seconds.

a Make t the subject of the formula. Consider $t > 0$ only.

b Find the time taken for an object accelerating at 8 cm s^{-2} to travel 10 m.

9 The relationship between the object and image distances (in cm) for a concave mirror can be written as $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ where f is the focal length, u is the object distance and v is the image distance.



a Make v the subject of the formula.

b Given a focal length of 8 cm, find the image distance for the following object distances: **i** 50 cm **ii** 30 cm.

10 According to Einstein's theory of relativity, the mass of a particle is given by the formula $m = \dfrac{m_0}{\sqrt{1 - \left(\dfrac{v}{c}\right)^2}}$, where m_0 is the mass of the particle at rest, v is the speed of the particle, and c is the speed of light.

a Make v the subject of the formula given $v > 0$.

b Find the speed necessary to increase the mass of a particle to three times its rest mass, i.e., $m = 3m_0$. Give the value for v as a fraction of c.

c A cyclotron increased the mass of an electron to $30m_0$. With what velocity must the electron have been travelling, given $c = 3 \times 10^8\ \text{m s}^{-1}$?

J ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

To add or subtract algebraic fractions, we combine them into a single fraction with the **least common denominator** (LCD).

For example, $\frac{x-1}{3} - \frac{x+3}{2}$ has LCD of 6, so we write each fraction with denominator 6.

EXERCISE J

Example 21

Write as a single fraction: **a** $2 + \frac{3}{x}$ **b** $\frac{x-1}{3} - \frac{x+3}{2}$

a
$$2 + \frac{3}{x}$$
$$= 2\left(\frac{x}{x}\right) + \frac{3}{x}$$
$$= \frac{2x+3}{x}$$

b
$$\frac{x-1}{3} - \frac{x+3}{2}$$
$$= \frac{2}{2}\left(\frac{x-1}{3}\right) - \frac{3}{3}\left(\frac{x+3}{2}\right)$$
$$= \frac{2(x-1) - 3(x+3)}{6}$$
$$= \frac{2x - 2 - 3x - 9}{6}$$
$$= \frac{-x - 11}{6}$$

1 Write as a single fraction:

a $3 + \frac{x}{5}$ **b** $1 + \frac{3}{x}$ **c** $3 + \frac{x-2}{2}$

d $3 - \frac{x-2}{4}$ **e** $\frac{2+x}{3} + \frac{x-4}{5}$ **f** $\frac{2x+5}{4} - \frac{x-1}{6}$

Example 22

Write $\frac{3x+1}{x-2} - 2$ as a single fraction.

$$\frac{3x+1}{x-2} - 2$$
$$= \left(\frac{3x+1}{x-2}\right) - 2\left(\frac{x-2}{x-2}\right) \quad \{\text{the LCD is } (x-2)\}$$
$$= \frac{(3x+1) - 2(x-2)}{x-2}$$
$$= \frac{3x + 1 - 2x + 4}{x-2}$$
$$= \frac{x+5}{x-2}$$

2 Write as a single fraction:

a $1+\dfrac{3}{x+2}$ **b** $-2+\dfrac{3}{x-4}$ **c** $-3-\dfrac{2}{x-1}$

d $\dfrac{2x-1}{x+1}+3$ **e** $3-\dfrac{x}{x+1}$ **f** $-1+\dfrac{4}{1-x}$

3 Write as a single fraction:

a $\dfrac{3x}{2x-5}+\dfrac{2x+5}{x-2}$ **b** $\dfrac{1}{x-2}-\dfrac{1}{x-3}$

c $\dfrac{5x}{x-4}+\dfrac{3x-2}{x+4}$ **d** $\dfrac{2x+1}{x-3}-\dfrac{x+4}{2x+1}$

K CONGRUENCE AND SIMILARITY

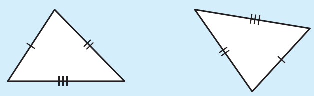
CONGRUENCE

Two triangles are **congruent** if they are identical in every respect apart from position. The triangles have the same shape and size.

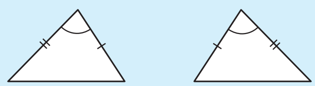
There are four acceptable tests for the **congruence of two triangles**.

Two triangles are congruent if one of the following is true:

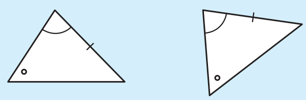
- corresponding sides are equal in length (**SSS**)



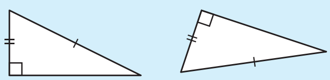
- two sides and the included angle are equal (**SAS**)



- two angles and a pair of corresponding sides are equal (**AAcorS**)

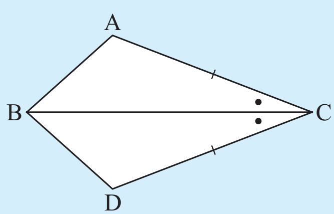


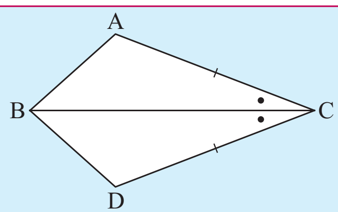
- for right angled triangles, the hypotenuse and one other pair of sides are equal (**RHS**).



Example 23

Explain why ΔABC and ΔDBC are congruent:





Δs ABC and DBC are congruent (SAS) as:

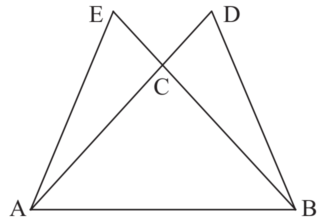
- AC = DC
- $A\widehat{C}B = D\widehat{C}B$, and
- [BC] is common to both.

If congruence can be proven then all corresponding lengths, angles and areas must be equal.

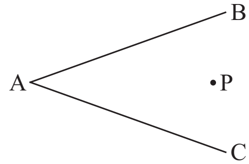
EXERCISE K.1

1 Triangle ABC is isosceles with AC = BC. [BC] and [AC] are produced to E and D respectively so that CE = CD.

Prove that AE = BD.

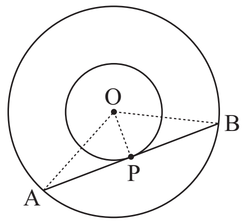


2 Point P is equidistant from both [AB] and [AC]. Use congruence to show that P lies on the bisector of $B\widehat{A}C$.



3 Two concentric circles are drawn. At P on the inner circle, a tangent is drawn which meets the other circle at A and B.

Use triangle congruence to prove that P is the midpoint of [AB].

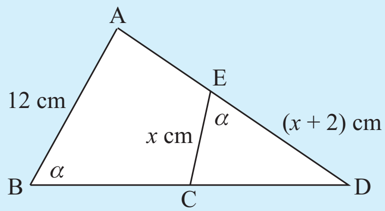


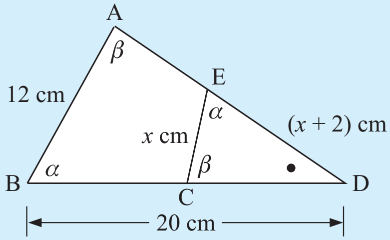
SIMILARITY

Two triangles are **similar** if one is an enlargement of the other.
Similar triangles are **equiangular**, and have corresponding sides in the same **ratio**.

Example 24

Establish that a pair of triangles is similar and find x if BD = 20 cm:





α	β	$\bullet$	
-	$x+2$	x	small Δ
-	20	12	large Δ

The triangles are equiangular and hence similar.

$$\therefore \quad \frac{x+2}{20} = \frac{x}{12} \quad \{\text{sides in the same ratio}\}$$

$$\therefore \quad 12(x+2) = 20x$$

$$\therefore \quad 12x + 24 = 20x$$

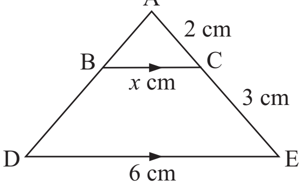
$$\therefore \quad 24 = 8x$$

$$\therefore \quad x = 3$$

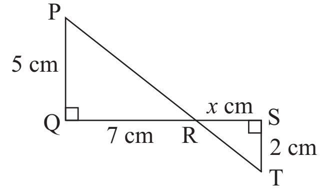
EXERCISE K.2

1 In each of the following, establish that a pair of triangles is similar, and hence find x:

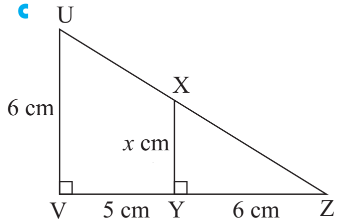
a



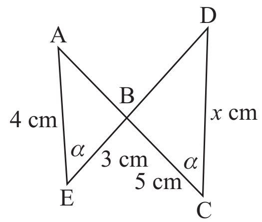
b



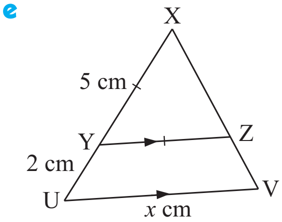
c



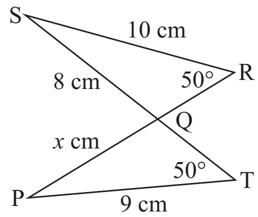
d



e



f



2 A father and son are standing side-by-side. The father is 1.8 m tall and casts a shadow 3.2 m long, while his son's shadow is 2.4 m long. How tall is the son?

L COORDINATE GEOMETRY

THE NUMBER PLANE

The position or location of any point in the **number plane** can be specified in terms of an **ordered pair** of numbers (x, y), where x is the **horizontal step** from a fixed point O, and y is the **vertical step** from O.

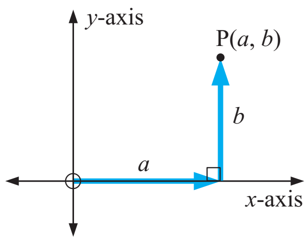
The point O is called the **origin**. Once O has been specified, we draw two perpendicular axes through it.

DEMO

The ***x*-axis** is horizontal and the ***y*-axis** is vertical.

The **number plane** is also known as either:

- the **2-dimensional plane**, or
- the **Cartesian plane**, named after **René Descartes**.



(a, b) is called an **ordered pair**, where a and b are the coordinates of the point.

a is called the ***x*-coordinate**.

b is called the ***y*-coordinate**.

THE DISTANCE FORMULA

If A(x_1, y_1) and B(x_2, y_2) are two points in a plane, then the distance between these points is given by **AB** $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 25

Find the distance between A(-2, 1) and B(3, 4).

A(-2, 1) B(3, 4)

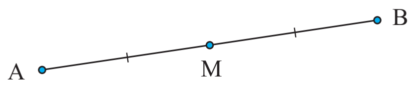
$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$

x_1 y_1 x_2 y_2

$$\begin{aligned} \text{AB} &= \sqrt{(3 - -2)^2 + (4 - 1)^2} \\ &= \sqrt{5^2 + 3^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \text{ units} \end{aligned}$$

THE MIDPOINT FORMULA

If M is halfway between points A and B then M is the **midpoint** of [AB].



If A(x_1, y_1) and B(x_2, y_2) are two points then the **midpoint** M of [AB] has coordinates $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

Example 26

Find the coordinates of the midpoint of [AB] for A(-1, 3) and B(4, 7).

The x-coordinate of the midpoint $= \dfrac{-1 + 4}{2} = \frac{3}{2} = 1\frac{1}{2}$

The y-coordinate of the midpoint $= \dfrac{3 + 7}{2} = 5$

$\therefore$ the midpoint of [AB] is $(1\frac{1}{2}, 5)$.

THE GRADIENT OR SLOPE OF A LINE

When looking at line segments drawn on a set of axes, it is clear that different line segments are inclined to the horizontal at different angles. Some appear to be steeper than others.

The **gradient** or **slope** of a line is a measure of its steepness.

If A is (x_1, y_1) and B is (x_2, y_2) then the **gradient** of [AB] is $\dfrac{y_2 - y_1}{x_2 - x_1}$.

Example 27

Find the gradient of the line through $(3, -2)$ and $(6, 4)$.

$(3, -2)$ $(6, 4)$
$\uparrow\ \uparrow$ $\uparrow\ \uparrow$
$x_1\ \ y_1$ $x_2\ \ y_2$

gradient $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - -2}{6 - 3} = 2$

Note:
- horizontal lines have a gradient of **0 (zero)**
- vertical lines have an **undefined** gradient
-  forward sloping lines have **positive** gradients
-  backward sloping lines have **negative** gradients
- **parallel lines** have equal gradients
- the gradients of **perpendicular lines** are *negative reciprocals*,

  i.e., if the gradients are m_1 and m_2 then $\boldsymbol{m_2 = \dfrac{-1}{m_1}}$ or $\boldsymbol{m_1 m_2 = -1}$.

  This is true except when the lines are parallel to the axes.

EQUATIONS OF LINES

The **equation of a line** states the connection between the x and y values for every point on the line, and only for points on the line

Equations of lines have various forms:

- All **vertical lines** have equations of the form $x = a$ where a is a constant.
- All **horizontal lines** have equations of the form $y = c$ where c is a constant.
- If a straight line has gradient m and passes through (a, b)

  then it has equation $\boldsymbol{\dfrac{y - b}{x - a} = m}$ or $\boldsymbol{y - b = m(x - a)}$ {**point-gradient** form}

  which can be rearranged into $\boldsymbol{y = mx + c}$ {**gradient-intercept** form}

  or $\boldsymbol{Ax + By = C}$ {**general** form}

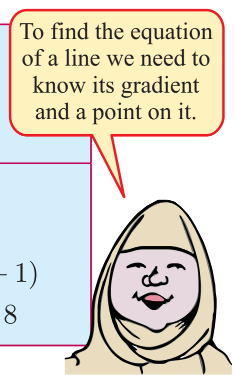
Example 28

Find, in gradient-intercept form, the equation of the line through (1, 3) with a slope of 5.

The equation of the line is $\dfrac{y - 3}{x - -1} = 5$ i.e., $\dfrac{y - 3}{x + 1} = 5$

$\therefore\ y - 3 = 5(x + 1)$

$\therefore\ y = 5x + 8$



Example 29

Find, in general form, the equation of the line through $(1, -5)$ and $(5, -2)$.

The slope $= \dfrac{-2 - -5}{5 - 1}$

$= \frac{3}{4}$

So, the equation is $\dfrac{y - -2}{x - 5} = \dfrac{3}{4}$

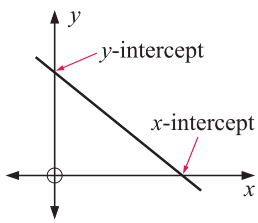
$\therefore \dfrac{y + 2}{x - 5} = \dfrac{3}{4}$

$\therefore 4y + 8 = 3x - 15$

$\therefore 3x - 4y = 23$

AXES INTERCEPTS

Axes **intercepts** are the x- and y-values where a graph cuts the coordinate axes.



The x-intercept is found by letting $y = 0$.
The y-intercept is found by letting $x = 0$.

Example 30

For the line with equation $2x - 3y = 12$, find the axes intercepts.

When $x = 0$, $-3y = 12$
$\therefore y = -4$

When $y = 0$, $2x = 12$
$\therefore x = 6$

So, the y-intercept is -4 and the x-intercept is 6.

DOES A POINT LIE ON A LINE?

A point lies on a line if its coordinates satisfy the equation of the line.

Example 31

Does $(3, -2)$ lie on the line with equation $5x - 2y = 20$?

Substituting $(3, -2)$ into $5x - 2y = 20$ gives
$5(3) - 2(-2) = 20$
i.e., $19 = 20$ which is false

$\therefore$ $(3, -2)$ does not lie on the line.

WHERE GRAPHS MEET

Example 32

Use graphical methods to find where the lines $x + y = 6$ and $2x - y = 6$ meet.

For $x + y = 6$:

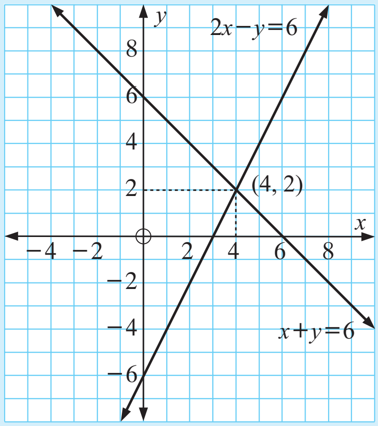
when $x = 0$, $y = 6$
when $y = 0$, $x = 6$

x	0	6
y	6	0

For $2x - y = 6$:

when $x = 0$, $-y = 6$,
$\therefore$ $y = -6$
when $y = 0$, $2x = 6$,
$\therefore$ $x = 3$

x	0	3
y	-6	0

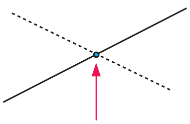


The graphs meet at (4, 2).

Check: $4 + 2 = 6$ ✓ and $2 \times 4 - 2 = 6$ ✓

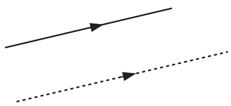
There are three possible situations which may occur. These are:

Case 1:



The lines meet in a single **point of intersection**.

Case 2:



The lines are **parallel** and **never meet**. There is no point of intersection.

Case 3:

The lines are **coincident**. There are infinitely many points of intersection.

INVESTIGATION 2 FINDING WHERE LINES MEET USING TECHNOLOGY

Graphing packages and **graphics calculators** can be used to plot graphs and hence find their point(s) of intersection. This can be useful if the solutions are not integer values.

One downside of using graphics calculators is that most require the equations to be entered in the form $y = f(x)$. So, if the equation of a straight line is given in **general** form, it must be rearranged into **gradient-intercept** form.

GRAPHING PACKAGE

Suppose we wish to use technology to find the point of intersection of $4x + 3y = 10$ and $x - 2y = -3$:

If you are using the **graphing package**, click on the icon to open the package and enter the two equations.

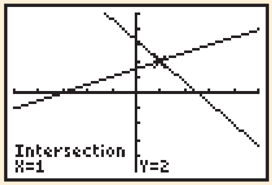
If you are using a **graphics calculator**, follow the following steps:

Step 1: We **rearrange** each equation into the form $y = mx + c$:

$$4x + 3y = 10 \qquad\qquad x - 2y = -3$$
$$\therefore\quad 3y = -4x + 10 \qquad\qquad \therefore\quad -2y = -x - 3$$
$$\therefore\quad y = -\tfrac{4}{3}x + \tfrac{10}{3} \qquad\qquad \therefore\quad y = \frac{x}{2} + \frac{3}{2}$$

Step 2: Enter the functions $\text{Y}_1 = -4\text{X}/3 + 10/3$ and $\text{Y}_2 = \text{X}/2 + 3/2$.

Step 3: Draw the **graphs** of the functions on the same set of axes. You may have to change the viewing **window**.



TI

C

Step 4: Use the built in functions to calculate the point of **intersection**. Thus, the point of intersection is (1, 2).

What to do:

1 Use technology to find the point of intersection of:

a $y = x + 4$
$5x - 3y = 0$

b $x + 2y = 8$
$y = 7 - 2x$

c $x - y = 5$
$2x + 3y = 4$

d $2x + y = 7$
$3x - 2y = 1$

e $y = 3x - 1$
$3x - y = 6$

f $y = -\dfrac{2x}{3} + 2$
$2x + 3y = 6$

2 Comment on the use of technology to find the point(s) of intersection in **1 e** and **1 f**.

EXERCISE L

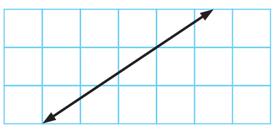
1 Use the distance formula to find the distance between the following pairs of points:

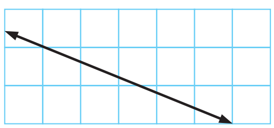
a A(1, 3) and B(4, 5)
b O(0, 0) and C(3, −5)
c P(5, 2) and Q(1, 4)
d S(0, −3) and T(−1, 0).

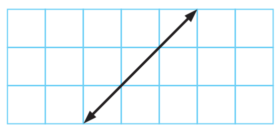
2 Find the midpoint of [AB] for:

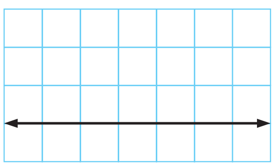
a A(3, 6) and B(1, 0)
b A(5, 2) and B(−1, −4)
c A(7, 0) and B(0, 3)
d A(5, −2) and B(−1, −3).

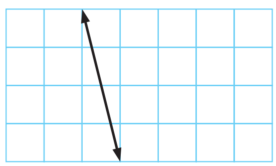
3 By finding a y-step and an x-step, determine the slope of each of the following lines:

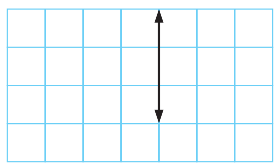
a


b


c


d


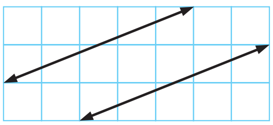
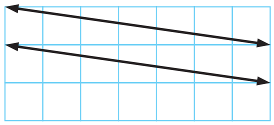
e


f


4 Find the gradient (slope) of the line passing through:

a (2, 3) and (4, 7)
b (3, 2) and (5, 8)
c (−1, 2) and (−1, 5)
d (4, −3) and (−1, −3)
e (0, 0) and (−1, 4)
f (3, −1) and (−1, −2).

5 Classify the following pairs of lines as parallel, perpendicular or neither. Give reasons for your answers.

a

b

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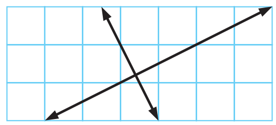
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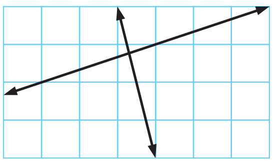
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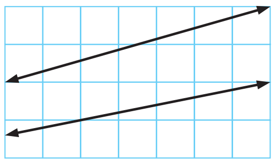
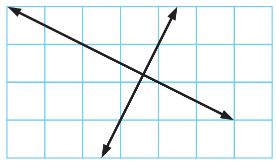

f


6 State the slope of the line which is perpendicular to the line with gradient (slope):

a $\frac{3}{4}$ **b** $\frac{11}{3}$ **c** 4 **d** $-\frac{1}{3}$ **e** −5 **f** 0

7 Find, in gradient-intercept form, the equation of the line through:

a (4, 1) with slope 2
b (1, 2) with slope −2
c (5, 0) with slope 3
d (−1, 7) with slope −3
e (1, 5) with slope −4
f (2, 7) with slope 1.

8 Find, in general form, the equation of the line through:

a (2, 1) with slope $\frac{3}{2}$
b (1, 4) with slope $-\frac{3}{2}$
c (4, 0) with slope $\frac{1}{3}$
d (0, 6) with slope −4
e (−1, −3) with slope 3
f (4, −2) with slope $-\frac{4}{9}$.

9 Find the equations of the lines through:

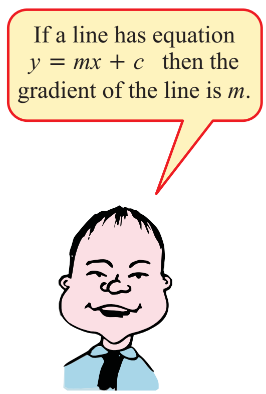
a (0, 1) and (3, 2)
b (1, 4) and (0, −1)
c (2, −1) and (−1, −4)
d (0, −2) and (5, 2)
e (3, 2) and (−1, 0)
f (−1, −1) and (2, −3)

10 Find the equations of the lines through:

a (3, −2) and (5, −2)
b (6, 7) and (6, −11)
c (−3, 1) and (−3, −3)

11 Copy and complete:

	Equation of line	*Gradient*	*x-intercept*	*y-intercept*
a	$2x-3y=6$			
b	$4x+5y=20$			
c	$y=-2x+5$			
d	$x=8$			
e	$y=5$			
f	$x+y=11$			
g	$4x+y=8$			
h	$x-3y=12$			



12 **a** Does (3, 4) lie on the line with equation $3x - 2y = 1$?

b Does (−2, 5) lie on the line with equation $5x + 3y = -5$?

c Does $(6, -\frac{1}{2})$ lie on the line $3x - 8y = 22$?

13 Use graphical methods to find where the following lines meet:

a $x + 2y = 8$, $y = 2x - 6$

b $y = -3x - 3$, $3x - 2y = -12$

c $3x + y = -3$, $2x - 3y = -24$

d $2x - 3y = 8$, $3x + 2y = 12$

e $x + 3y = 10$, $2x + 6y = 11$

f $5x + 3y = 10$, $10x + 6y = 20$

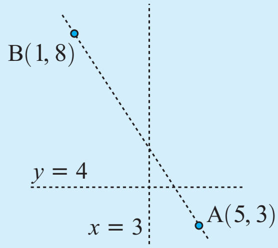
Example 33

A straight road is to pass through the points A(5, 3) and B(1, 8).

a Find where this road meets the road given by:
  i $x = 3$ **ii** $y = 4$

b If we wish to refer to the points on the road (AB) *between* A and B, how can we indicate this?

c Does C(23, −20) lie on the road?



First we must find the equation of the line representing the road.

Its gradient is $m = \dfrac{3-8}{5-1} = -\frac{5}{4}$

So, its equation is
$$\frac{y-3}{x-5} = -\frac{5}{4}$$
$$\therefore\ 4(y-3) = -5(x-5)$$
$$\therefore\ 4y - 12 = -5x + 25$$
$$\therefore\ 5x + 4y = 37$$

a **i** When $x = 3$, $5(3) + 4y = 37$
$\therefore\ 15 + 4y = 37$
$\therefore\ 4y = 22$
$\therefore\ y = 5\frac{1}{2}$

$\therefore$ meet at $(3, 5\frac{1}{2})$.

ii When $y = 4$, $5x + 4(4) = 37$
$\therefore\ 5x + 16 = 37$
$\therefore\ 5x = 21$
$\therefore\ x = 4.2$

$\therefore$ they meet at (4.2, 4).

b We restrict the possible x-values to $1 \leqslant x \leqslant 5$.

c If C(23, −20) lies on the line, its coordinates must satisfy the line's equation.

Now LHS $= 5(23) + 4(-20)$
$= 115 - 80$
$= 35$
$\neq 37$ $\therefore$ C does not lie on the road.

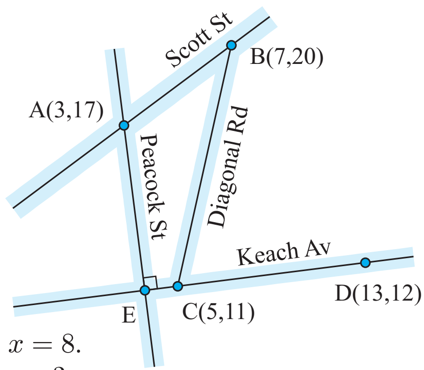
14 Find the equation of the:

- **a** horizontal line through $(3,\ -4)$
- **b** vertical line with x-intercept 5
- **c** vertical line through $(-1,\ -3)$
- **d** horizontal line with y-intercept 2
- **e** x-axis
- **f** y-axis.

15 Find the equation of the line which is:

- **a** through A$(-1,\ 4)$ and with gradient $\frac{3}{4}$
- **b** through P$(2,\ -5)$ and Q$(7,\ 0)$
- **c** parallel to the line with equation $y = 3x - 2$ and passes through $(0,\ 0)$
- **d** parallel to the line with equation $2x + 3y = 8$ and passes through $(-1,\ 7)$
- **e** perpendicular to the line with equation $y = -2x + 5$ and passes through $(3,\ -1)$
- **f** perpendicular to the line with equation $3x - y = 11$ and passes through $(-2,\ 5)$.

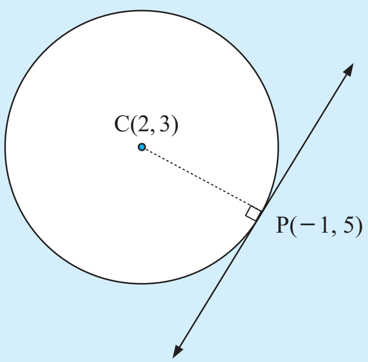
16 A is the town hall on Scott Street and D is a Post Office on Keach Avenue. Diagonal Road intersects Scott Street at B and Keach Avenue at C.



- **a** Find the equation of Keach Avenue.
- **b** Find the equation of Peacock Street.
- **c** Find the equation of Diagonal Road. (Be careful!)
- **d** Plunkit Street lies on the map reference line $x = 8$. Where does Plunkit Street intersect Keach Avenue?

Example 34

Find the equation of the tangent to the circle with centre $(2,\ 3)$ at the point $(-1,\ 5)$.



The gradient of [CP] is $\dfrac{3-5}{2-(-1)} = \dfrac{-2}{3} = -\frac{2}{3}$

$\therefore$ the gradient of the tangent at P is $\frac{3}{2}$.

Since the tangent passes through $(-1,\ 5)$,

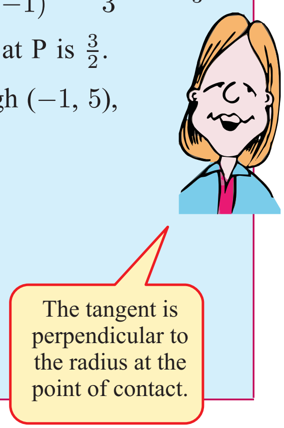
its equation is

$$\frac{y-5}{x--1} = \frac{3}{2}$$

$\therefore\ 2(y-5) = 3(x+1)$

$\therefore\ 2y - 10 = 3x + 3$

$\therefore\ 3x - 2y = -13$

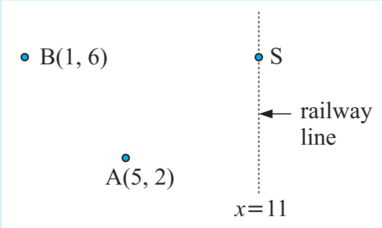


17 Find the equation of the tangent to the circle with centre:

- **a** $(0,\ 2)$ at $(-1,\ 5)$
- **b** $(3,\ -1)$ at $(-1,\ 1)$
- **c** $(2,\ -2)$ at $(5,\ -2)$.

Example 35

Mining towns are situated at B(1, 6) and A(5, 2). Where should the railway siding S be located so that ore trucks from either A or B would travel equal distances to a railway line with equation $x = 11$?

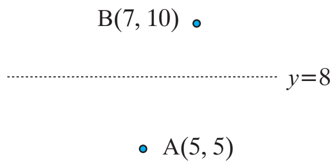


Suppose S has the coordinates $(11, a)$.

$$\text{Now} \quad BS = AS$$
$$\therefore \quad \sqrt{(11-1)^2+(a-6)^2} = \sqrt{(11-5)^2+(a-2)^2}$$
$$\therefore \quad 10^2+(a-6)^2 = 6^2+(a-2)^2 \qquad \{\text{squaring both sides}\}$$
$$\therefore \quad 100+a^2-12a+36 = 36+a^2-4a+4$$
$$\therefore \quad -12a+4a = 4-100$$
$$\therefore \quad -8a = -96$$
$$\therefore \quad a = 12$$

So, the railway siding should be located at (11, 12).

18 A(5, 5) and B(7, 10) are houses and $y = 8$ is a gas pipeline. Where should the one outlet from the pipeline be placed so that it is the same distance from both houses?

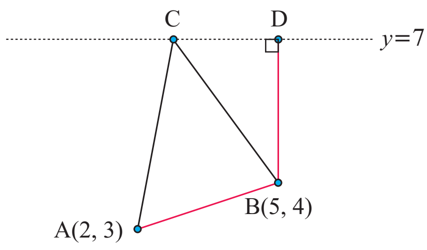


19 (CD) is a water pipeline. A and B are two towns. A pumping station is to be located on the pipeline to pump water to A and B. Each town is to pay for their own service pipes and they insist on equality of costs.

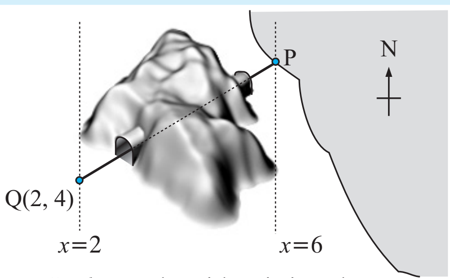
a Where should C be located to ensure equality of costs?

b What is the total length of service pipe required?

c If the towns agree to pay equal amounts, would it be cheaper to install the service pipeline from D to B to A?



Example 36



Scale: each grid unit is 1 km.

A tunnel through the mountains connects town Q(2, 4) to the port at P.

P is on grid reference $x = 6$ and the distance between the town and the port is 5 km.

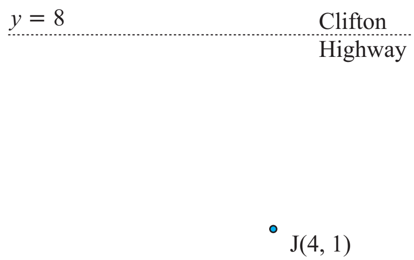
Assuming the diagram is reasonably accurate, what is the horizontal grid reference of the port?

Suppose P is at $(6,\ a)$.

$$\begin{aligned} \text{Now}\quad PQ &= 5 \\ \therefore\quad \sqrt{(6-2)^2+(a-4)^2} &= 5 \\ \therefore\quad \sqrt{16+(a-4)^2} &= 5 \\ \therefore\quad 16+(a-4)^2 &= 25 \\ \therefore\quad (a-4)^2 &= 9 \\ \therefore\quad a-4 &= \pm 3 \\ \therefore\quad a &= 4 \pm 3 = 7 \text{ or } 1 \end{aligned}$$

But from the diagram, P is further north than Q $\quad\therefore\quad a > 4$

So, P is at $(6,\ 7)$ and the horizontal grid reference is $\ y = 7$.

20 

Jason's girlfriend lives in a house on Clifton Highway which has equation $y = 8$. The distance 'as the crow flies' from Jason's house to his girlfriend's house is 11.73 km. If Jason lives at $(4,\ 1)$, what are the coordinates of his girlfriend's house?

Scale: 1 unit $\equiv$ 1 km.

21 **a** A circle has centre $(a,\ b)$ and radius r units. $P(x,\ y)$ moves on the circle. Show that $\ (x-a)^2+(y-b)^2 = r^2$.

b Find the equation of the circle with:

- **i** centre $(4,\ 3)$ and radius 5 units
- **ii** centre $(-1,\ 5)$ and radius 2 units
- **iii** centre $(0,\ 0)$ and radius 10 units
- **iv** ends of a diameter $(-1,\ 5)$ and $(3,\ 1)$.

22 Find the centre and radius of the circle:

a $(x-1)^2+(y-3)^2 = 4$ **b** $x^2+(y+2)^2 = 16$ **c** $x^2+y^2 = 7$

23 Consider the circle with equation $\ (x-2)^2+(y+3)^2 = 20$.

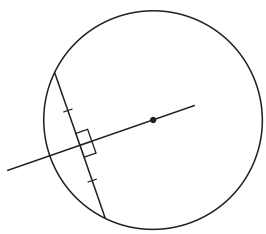
a State the circle's centre and radius.

b Show that $(4,\ 1)$ lies on the circle.

c Find the equation of the tangent to the circle at the point $(4,\ 1)$.

24 Recall that the perpendicular bisector of a chord of a circle passes through the centre of the circle.

Find the centre of a circle passing through points $P(5,\ 7)$, $Q(7,\ 1)$ and $R(-1,\ 5)$ by finding the perpendicular bisectors of [PQ] and [QR] and solving them simultaneously.



EXERCISE A

1 **a** $\sqrt{15}$ **b** 3 **c** 4 **d** 12 **e** 42 **f** $\sqrt{6}$ **g** $\sqrt{2}$ **h** $\sqrt{6}$

2 **a** $5\sqrt{2}$ **b** $-\sqrt{2}$ **c** $2\sqrt{5}$ **d** $8\sqrt{5}$ **e** $-2\sqrt{5}$
f $9\sqrt{3}$ **g** $-3\sqrt{6}$ **h** $3\sqrt{2}$

3 **a** $2\sqrt{2}$ **b** $2\sqrt{3}$ **c** $2\sqrt{5}$ **d** $4\sqrt{2}$ **e** $3\sqrt{3}$ **f** $3\sqrt{5}$
g $4\sqrt{3}$ **h** $3\sqrt{6}$ **i** $5\sqrt{2}$ **j** $4\sqrt{5}$ **k** $4\sqrt{6}$ **l** $6\sqrt{3}$

4 **a** $2\sqrt{3}$ **b** $8\sqrt{2}$ **c** $5\sqrt{6}$ **d** $10\sqrt{3}$ **e** $3\sqrt{3}$ **f** $-\sqrt{2}$

5 **a** $\frac{\sqrt{2}}{2}$ **b** $2\sqrt{3}$ **c** $\frac{7\sqrt{2}}{2}$ **d** $2\sqrt{5}$ **e** $5\sqrt{2}$ **f** $3\sqrt{6}$
g $4\sqrt{3}$ **h** $\frac{5\sqrt{7}}{7}$ **i** $2\sqrt{7}$ **j** $\sqrt{6}$

EXERCISE B

1 **a** 2.59×10^2 **b** 2.59×10^5 **c** 2.59×10^0
d 2.59×10^{-1} **e** 2.59×10^{-4} **f** 4.07×10^1
g 4.07×10^3 **h** 4.07×10^{-2} **i** 4.07×10^5
j 4.07×10^8 **k** 4.07×10^{-5}

2 **a** 1.495×10^{11} m **b** 3×10^{-4} mm **c** 1×10^{-3} mm
d 1.5×10^7 °C **e** 3×10^5

3 **a** 4000 **b** 500 **c** 2100 **d** 78 000 **e** 380 000
f 86 **g** 43 300 000 **h** 60 000 000

4 **a** 0.004 **b** 0.05 **c** 0.0021 **d** 0.000 78
e 0.000 038 **f** 0.86 **g** 0.000 000 433 **h** 0.000 000 6

5 **a** 0.000 000 9 m **b** 6 130 000 000 **c** 100 000 light years
d 0.000 01 mm

6 **a** 1.64×10^{10} **b** 4.12×10^{-3} **c** 5.27×10^{-18}
d 1.36×10^2 **e** 2.63×10^{-6} **f** 1.73×10^9

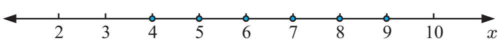
7 **a** 1.30×10^5 km **b** 9.07×10^5 km **c** 9.47×10^7 km

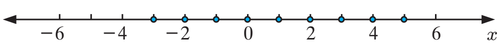
8 **a** 1.8×10^{10} m **b** 2.59×10^{13} m **c** 9.47×10^{15} m

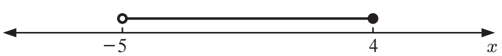
EXERCISE C

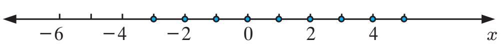
1 **a** The set of all real x such that x is greater than 5.
b The set of all integers x such that x is less than or equal to 3.
c The set of all y such that y lies between 0 and 6.
d The set of all integers x such that x is greater than or equal to 2, but less than or equal to 4. x is 2, 3 or 4.
e The set of all t such that t lies between 1 and 5.
f The set of all n such that n is less than 2 or greater than or equal to 6

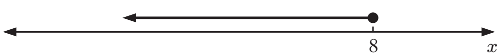
2 **a** $\{x \mid x > 2\}$ **b** $\{x \mid 1 < x \leqslant 5\}$
c $\{x \mid x \leqslant -2 \text{ or } x \geqslant 3\}$ **d** $\{x \mid -1 \leqslant x \leqslant 3, x \in \mathbb{Z}\}$
e $\{x \mid 0 \leqslant x \leqslant 5, x \in \mathbb{Z}\}$ **f** $\{x \mid x < 0\}$

3 **a**


b


c


d


e


EXERCISE D

1 **a** $10x - 10$ **b** $9x$ **c** $5x + 5y$ **d** $8 - 8x$ **e** $12ab$
f cannot be simplified

2 **a** $22x + 35$ **b** $16 - 6x$ **c** $4a - 3b$
d $3x^3 - 16x^2 + 11x - 1$

3 **a** $18x^3$ **b** $\frac{a}{3b}$ **c** $4x^2$ **d** $24a^{10}$

EXERCISE E

1 **a** $x = 10$ **b** $x > 6$ **c** $x = \frac{4}{5}$ **d** $x = 51$ **e** $x < -10$
f $x = 14$ **g** $x \leqslant -9$ **h** $x = 18$ **i** $x = \frac{2}{3}$

2 **a** $x = 5, y = 2$ **b** $x = \frac{22}{3}, y = \frac{8}{3}$ **c** $x = -2, y = 5$
d $x = \frac{45}{11}, y = -\frac{18}{11}$ **e** no solution
f $x = 66, y = -84$

EXERCISE F

1 **a** 16 **b** −6 **c** 16 **d** 18 **e** −2 **f** 2

2 **a** 2 **b** 3 **c** 6 **d** 6 **e** 5 **f** −1 **g** 1 **h** 5
i 4 **j** 4 **k** 2 **l** 2

3 **a** $x = \pm 3$ **b** no solution **c** $x = 0$ **d** $x = 4$ or -2
e $x = -1$ or 7 **f** no solution **g** $x = 1$ or $\frac{1}{3}$
h $x = 0$ or 3 **i** $x = -2$ or $\frac{14}{5}$

EXERCISE G

1 **a** $2x^2 + 5x + 3$ **b** $3x^2 + 10x + 8$ **c** $10x^2 + x - 2$
d $3x^2 + x - 10$ **e** $-6x^2 + 17x + 14$ **f** $-6x^2 - 13x + 5$
g $15x^2 + 11x - 12$ **h** $15x^2 - 11x + 2$ **i** $2x^2 - 17x + 21$
j $4x^2 - 16x + 15$ **k** $-x^2 - 3x - 2$ **l** $-4x^2 - 2x + 6$

2 **a** $x^2 - 36$ **b** $x^2 - 64$ **c** $4x^2 - 1$ **d** $9x^2 - 4$
e $16x^2 - 25$ **f** $25x^2 - 9$ **g** $9 - x^2$ **h** $49 - x^2$
i $49 - 4x^2$ **j** $x^2 - 2$ **k** $x^2 - 5$ **l** $4x^2 - 3$

3 **a** $x^2 + 10x + 25$ **b** $x^2 + 14x + 49$ **c** $x^2 - 4x + 4$
d $x^2 - 12x + 36$ **e** $x^2 + 6x + 9$ **f** $x^2 + 10x + 25$
g $x^2 - 22x + 121$ **h** $x^2 - 20x + 100$ **i** $4x^2 + 28x + 49$
j $9x^2 + 12x + 4$ **k** $4x^2 - 20x + 25$ **l** $9x^2 - 42x + 49$

4 **a** $y = 2x^2 + 10x + 12$ **b** $y = 3x^2 - 6x + 7$
c $y = -x^2 + 6x + 7$ **d** $y = -x^2 - 4x - 15$
e $y = 4x^2 - 24x + 20$ **f** $y = -\frac{1}{2}x^2 - 4x - 14$
g $y = -5x^2 + 35x - 30$ **h** $y = \frac{1}{2}x^2 + 2x - 4$
i $y = -\frac{5}{2}x^2 + 20x - 40$

5 **a** $2x^2 + 12x + 19$ **b** $3x^2 + 3x - 16$ **c** $-x^2 + 6x - 6$
d $-x^2 - x + 25$ **e** $2x^2 - 16x + 33$ **f** $-3x + 4$
g $7x + 8$ **h** $7x^2 + 18x + 12$ **i** $-x^2 + 19x - 32$
j $7x^2 - 16x + 2$

EXERCISE H

1 **a** $3x(x+3)$ **b** $x(2x+7)$ **c** $2x(2x-5)$ **d** $3x(2x-5)$
e $(3x-5)(3x+5)$ **f** $(4x+1)(4x-1)$ **g** $2(x-2)(x+2)$
h $3(x+\sqrt{3})(x-\sqrt{3})$ **i** $4(x+\sqrt{5})(x-\sqrt{5})$ **j** $(x-4)^2$
k $(x-5)^2$ **l** $2(x-2)^2$ **m** $(4x+5)^2$ **n** $(3x+2)^2$
o $(x-11)^2$

2 **a** $(x+8)(x+1)$ **b** $(x+4)(x+3)$ **c** $(x-9)(x+2)$
d $(x+7)(x-3)$ **e** $(x-6)(x-3)$ **f** $(x+3)(x-2)$
g $-(x-2)(x+1)$ **h** $3(x-11)(x-3)$ **i** $-2(x+1)^2$
j $2(x+5)(x-2)$ **k** $2(x-8)(x+3)$ **l** $-2(x-6)(x-1)$
m $-3(x-1)^2$ **n** $-(x+1)^2$ **o** $-5(x-4)(x+2)$

3 **a** $(2x-3)(x+4)$ **b** $(3x+1)(x-2)$ **c** $(7x-2)(x-1)$
d $(3x-2)(2x+1)$ **e** $(2x-3)(2x+1)$ **f** $(5x-3)(2x+1)$
g $(2x+1)(x-6)$ **h** $(3x+7)(x-4)$ **i** $(4x+3)(2x-1)$
j $(5x+3)(2x-3)$ **k** $(3x-1)(x+8)$ **l** $(3x+2)(2x+1)$
m $-2(2x+3)(x-1)$ **n** $(6x+1)(2x-3)$
o $-3(2x+7)(x-2)$ **p** $-(3x-2)(3x-5)$
q $(4x-9)(2x+3)$ **r** $(4x+3)(3x+1)$

s $(6x+1)(2x+3)$ **t** $(5x-4)(3x-2)$
u $(7x+5)(2x-3)$

4 **a** $(x+4)(2x+1)$ **b** $(2-x)(5-3x)$ **c** $3(x+2)(2x+7)$
d $4(x+5)(2x+11)$ **e** $2x(x+3)$ **f** $5(x+3)$
g $(x-2)(3x+26)$ **h** $(x-1)(2x-1)$

5 **a** $(x+7)(x-1)$ **b** $(x+1)(3-x)$ **c** $12(x+1)$
d $-4x(x+4)$ **e** $(3x+2)(x+4)$ **f** $h(2x+h)$
g $-12(x+1)$ **h** $-5(3x-4)(x-4)$ **i** $-3(x+9)(5x+9)$

EXERCISE I

1 **a** $x=b-a$ **b** $x=\frac{b}{a}$ **c** $x=\frac{d-a}{2}$ **d** $x=t-c$
e $x=\frac{20-2y}{5}$ **f** $x=\frac{12-3y}{2}$ **g** $x=\frac{d-3y}{7}$
h $x=\frac{c-by}{a}$ **i** $x=\frac{y-c}{m}$

2 **a** $z=\frac{b}{ac}$ **b** $z=\frac{a}{d}$ **c** $z=\frac{2d}{3}$

3 **a** $a=\frac{F}{m}$ **b** $r=\frac{C}{2\pi}$ **c** $d=\frac{V}{lh}$ **d** $K=\frac{b}{A}$

4 **a** $r=\sqrt{\frac{A}{\pi}}$ **b** $x=\sqrt[5]{aN}$ **c** $r=\sqrt[3]{\frac{3V}{4\pi}}$ **d** $x=\sqrt[3]{\frac{n}{D}}$

5 **a** $a=d^2n^2$ **b** $l=25T^2$ **c** $a=\pm\sqrt{b^2+c^2}$ **d** $l=\frac{gT^2}{4\pi^2}$
e $a=\frac{P}{2}-b$ **f** $h=\frac{A-\pi r^2}{2\pi r}$ **g** $r=\frac{E}{I}-R$ **h** $q=p-\frac{B}{A}$

6 **a** $a=\frac{d^2}{2kb}$ **b** 1.29 **7** **a** $r=\sqrt[3]{\frac{3V}{4\pi}}$ **b** 2.122 cm

8 **a** $t=\sqrt{\frac{2S}{a}}$ **b** 15.81 sec

9 **a** $v=\frac{uf}{u-f}$ **b** **i** 9.52 cm **ii** 10.9 cm

10 **a** $v=\sqrt{c^2\left(1-\frac{m_0^{\ 2}}{m^2}\right)}=\frac{c}{m}\sqrt{m^2-m_0^{\ 2}}$
b $v=\frac{\sqrt{8}}{3}c$ **c** $2.998\times10^8\ \text{m s}^{-1}$

EXERCISE J

1 **a** $\frac{15+x}{5}$ **b** $\frac{x+3}{x}$ **c** $\frac{x+4}{2}$ **d** $\frac{14-x}{4}$
e $\frac{8x-2}{15}$ **f** $\frac{4x+17}{12}$

2 **a** $\frac{x+5}{x+2}$ **b** $\frac{11-2x}{x-4}$ **c** $\frac{1-3x}{x-1}$ **d** $\frac{5x+2}{x+1}$
e $\frac{2x+3}{x+1}$ **f** $\frac{x+3}{1-x}$

3 **a** $\frac{7x^2-6x-25}{(2x-5)(x-2)}$ **b** $\frac{-1}{(x-2)(x-3)}$
c $\frac{8x^2+6x+8}{x^2-16}$ **d** $\frac{3x^2+3x+13}{(x-3)(2x+1)}$

EXERCISE K.1

1 **Hint:** Consider Δs AEC, BDC

2 **Hint:** Let M be on [AB] so that [PM] $\perp$ [AB].
Let N be on [AC] so that [PN] $\perp$ [AC].
Join [PM], [PN] and consider the two triangles formed.

EXERCISE K.2

1 **a** $x=2.4$ **b** $x=2.8$ **c** $x=3\frac{3}{11}$ **d** $x=6\frac{2}{3}$
e $x=7$ **f** $x=7.2$

2 1.35 m tall

EXERCISE L

1 **a** $\sqrt{13}$ units **b** $\sqrt{34}$ units **c** $\sqrt{20}$ units **d** $\sqrt{10}$ units

2 **a** (2, 3) **b** (2, −1) **c** $(3\frac{1}{2}, 1\frac{1}{2})$ **d** $(2, -2\frac{1}{2})$

3 **a** $\frac{2}{3}$ **b** $-\frac{2}{5}$ **c** 1 **d** 0 **e** −4 **f** undefined

4 **a** 2 **b** 3 **c** undefined **d** 0 **e** −4 **f** $\frac{1}{4}$

5 **a** parallel, slopes $\frac{2}{5}$ **b** parallel, slopes $-\frac{1}{7}$
c perpendicular, slopes $\frac{1}{2}$, −2 **d** neither, slopes −4, $\frac{1}{3}$
e neither, slopes $\frac{2}{7}$, $\frac{1}{5}$ **f** perpendicular, slopes 2, $-\frac{1}{2}$

6 **a** $-\frac{4}{3}$ **b** $-\frac{3}{11}$ **c** $-\frac{1}{4}$ **d** 3 **e** $\frac{1}{5}$ **f** undefined

7 **a** $y=2x-7$ **b** $y=-2x+4$ **c** $y=3x-15$
d $y=-3x+4$ **e** $y=-4x+9$ **f** $y=x+5$

8 **a** $3x-2y=4$ **b** $3x+2y=11$ **c** $x-3y=4$
d $4x+y=6$ **e** $3x-y=0$ **f** $4x+9y=-2$

9 **a** $x-3y=-3$ **b** $5x-y=1$ **c** $x-y=3$
d $4x-5y=10$ **e** $x-2y=-1$ **f** $2x+3y=-5$

10 **a** $y=-2$ **b** $x=6$ **c** $x=-3$

11

	Equation of line	*Slope*	*x-int.*	*y-int.*
a	$2x-3y=6$	$\frac{2}{3}$	3	−2
b	$4x+5y=20$	$-\frac{4}{5}$	5	4
c	$y=-2x+5$	−2	$\frac{5}{2}$	5
d	$x=8$	undef.	8	no y-int.
e	$y=5$	0	no x-int.	5
f	$x+y=11$	−1	11	11
g	$4x+y=8$	−4	2	8
h	$x-3y=12$	$\frac{1}{3}$	12	−4

12 **a** yes **b** no **c** yes

13 **a** (4, 2) **b** (−2, 3) **c** (−3, 6) **d** (4, 0)
e parallel lines do not meet **f** coincident lines

14 **a** $y=-4$ **b** $x=5$ **c** $x=-1$ **d** $y=2$
e $y=0$ **f** $x=0$

15 **a** $3x-4y=-19$ **b** $x-y=7$ **c** $y=3x$
d $2x+3y=19$ **e** $x-2y=5$ **f** $x+3y=13$

16 **a** $x-8y=-83$ **b** $8x+y=41$
c $9x-2y=23$ for $5\leqslant x\leqslant 7$ **d** $(8, 11\frac{3}{8})$

17 **a** $x-3y=-16$ **b** $2x-y=-3$ **c** $x=5$

18 $(4\frac{3}{4}, 8)$

19 **a** $(2\frac{1}{3}, 7)$ **b** 8.03 km **c** yes (6.16 km)

20 (13.41, 8) or (−5.41, 8)

21 **a** **Hint:** Use the distance formula to find the distance from the centre of the circle to point P.
b **i** $(x-4)^2+(y-3)^2=25$ **ii** $(x+1)^2+(y-5)^2=4$
iii $x^2+y^2=100$ **iv** $(x-1)^2+(y-3)^2=8$

22 **a** centre (1, 3), radius 2 units
b centre (0, −2), radius 4 units
c centre (0, 0), radius $\sqrt{7}$ units

23 **a** centre (2, −3), radius $\sqrt{20}$ units
b **Hint:** Substitute (4, 1) into equation of circle.
c $x+2y=6$

24 (3, 3)